AME Screening Exam – Spring 2016
Part 2: Major and Minor Areas

Friday, January 22, 2016

Name: ______________________________

Instructions:

The exam is closed book and closed notes.

Students are to solve 5 problems.

The 5 problems must be distributed across different areas according to the following rules.

- Three problems must be selected from the student's major area
- Two problems must be selected from an identified minor area

Majors and minors are to be selected from the following list of areas of specialization covered in this exam.

1. Combustion
2. Control Theory
3. Design
4. Dynamics and Vibrations
5. Fluid Dynamics

Note: The problems are arranged in the exam package according to the sequence of areas listed above.

Please identify your major and minor areas:

My Major Area is: ______________________

My Minor Area is: ______________________

This page must be returned as a cover page with your solutions.

You must also return the package of exam problems with your solutions.

Combustion

AME513 – Principles of Combustion

Screening Exam, January 22, 2016

AME513: Problem #1

1a. Consider the Lindemann mechanism for the following association reactions:

$$A + B \underset{k_2}{\overset{k_1}{\rightleftharpoons}} C^*$$

$$C^* + M \xrightarrow{k_3} C + M$$

The overall rate of production of C is given by:

$dC/dt = k_C\,[A]\,[B]$

Using the appropriate assumptions, derive k_C in the limit of low and high pressure limits in order to recover the Lindemann-like behavior

1b. The specific reaction rates for the branching reaction and termination reactions:

$H + O_2 \rightarrow OH + O$ (R1)

$H + O_2 + M \rightarrow HO_2 + M$ (R2)

are respectively:

$k_1 = A_1\, T^{a1} \exp(-E_{a1}/R^oT)$

$k_2 = A_2\, T^{a2} \exp(-E_{a2}/R^oT)$

For a given temperature T, derive a mathematical expression for the pressure at the second H_2/O_2 explosion limit. Please state your assumptions.

AME513: Problem #2

Experiments are performed in order to determine the laminar flame speeds, S_u^o, of propane/air mixtures at pressures ranging from 1 to 20 atm. It is known that S_u^o varies like:

$S_u^o \sim p^{(n/2-1)} \exp(-E_a/2R^oT_{ad})$, where

p is the pressure

n is the overall reaction order

E_a is the overall activation energy

T_{ad} is the adiabatic flame temperature

The S_u^o measurements are carried out for equivalence ratios $\phi = 0.6$ and $\phi = 1.0$.

Derive formulas that will allow for the determination of n and E_a based on certain assumptions.

Please evaluate the validity of those assumptions based on first-principle physical arguments and indicate whether they are more valid for the $\phi = 0.6$ or the $\phi = 1.0$ flames and why.

Spring 2016 Ph.D. screening exam question
AME 514

Consider a premixed flame propagating down a long tube. The adiabatic flame temperature (T_{ad}) of the mixture is 2000K, the constant-pressure specific heat (C_P) is 1400 J/kgK, the temperature-averaged mixture thermal diffusivity (α) is 1 cm^2/s, the temperature-averaged mixture kinematic viscosity (ν) is 0.7 cm^2/s, the characteristic reaction rate (ω) at 2000K is 1000 s^{-1}, and the overall activation energy is 160,000 J/mole. The Lewis numbers of fuel and O_2 are both close to 1. The universal gas constant ($\mathfrak{R}$) is 8.314 J/moleK.

(a) Estimate the adiabatic laminar burning velocity (S_l) of this mixture
(b) What is the minimum diameter tube through which this mixture can propagate without quenching?
(c) Estimate the minimum ignition energy of this mixture.
(d) Would coating the walls of the tube of part (b) with platinum catalyst affect the minimum tube diameter through which the mixture could propagate without quenching? Why or why not? If there were a change, would the minimum diameter increase or decrease? Explain.
(e) If this mixture were in a turbulent flow with turbulence intensity u' = 5 m/s and integral length scale (L_I) = 5 cm, what would the turbulent burning velocity (S_T) be? Assume that propagation is in the flamelet combustion regime, and use any model of turbulent burning velocity in this regime you want.
(f) In part (e), is this mixture really in the flamelet regime? Recall that Karlovitz number (Ka) for turbulent flames is defined by

$$Ka = 0.157 \mathrm{Re}_L^{-1/2}\left(\frac{u'}{S_L}\right)^2; \mathrm{Re}_L \equiv \frac{u'L_I}{\nu}$$

(g) If instead the mixture were in the distributed combustion regime, what would the turbulent burning velocity be according to Damköhler's hypothesis? Recall that the turbulent thermal diffusivity (α_T) can be estimated as $\alpha_T \approx 0.061\ \alpha\ \mathrm{Re}_L$.

Control Theory

Control Theory: Screening Examination
Spring 2016

January 22, 2016

The examination is closed notes and closed books, calculators are allowed.

1. Given Bode of a dynamic system. The dash line is the asymptotic approximation of the gain

   (a) Find the transfer function of the system $G(s)$.

   (b) Assuming that this system is in a unity feedback configuration, i.e. $H(s) = 1$

      i. What are the gain and phase margin of the system?

      ii. Is the closed-loop stable? Explain.

   (c) Assuming a proportional controller with gain K_p in the forward loop and a unity feedback configuration

      i. What needs to be K_p to have a gain margin of 10 db?

      ii. For what gains K_p will the closed loop system be unstable?

      iii. What needs to be K_p to have a phase margin of 10^o?

   (d) Assuming a integral controller with gain $\frac{K_I}{s}$ in the forward loop and a unity feedback configuration. Can the closed-loop system be stabilized for some values of the gain $K_I > 0$?

   (e) Assume that a time-delay T is present in the forward loop, i.e. if $G(s)$ is multiplied by e^{-Ts}

      i. If $T = 1\ s$ what is the phase margin of the system?

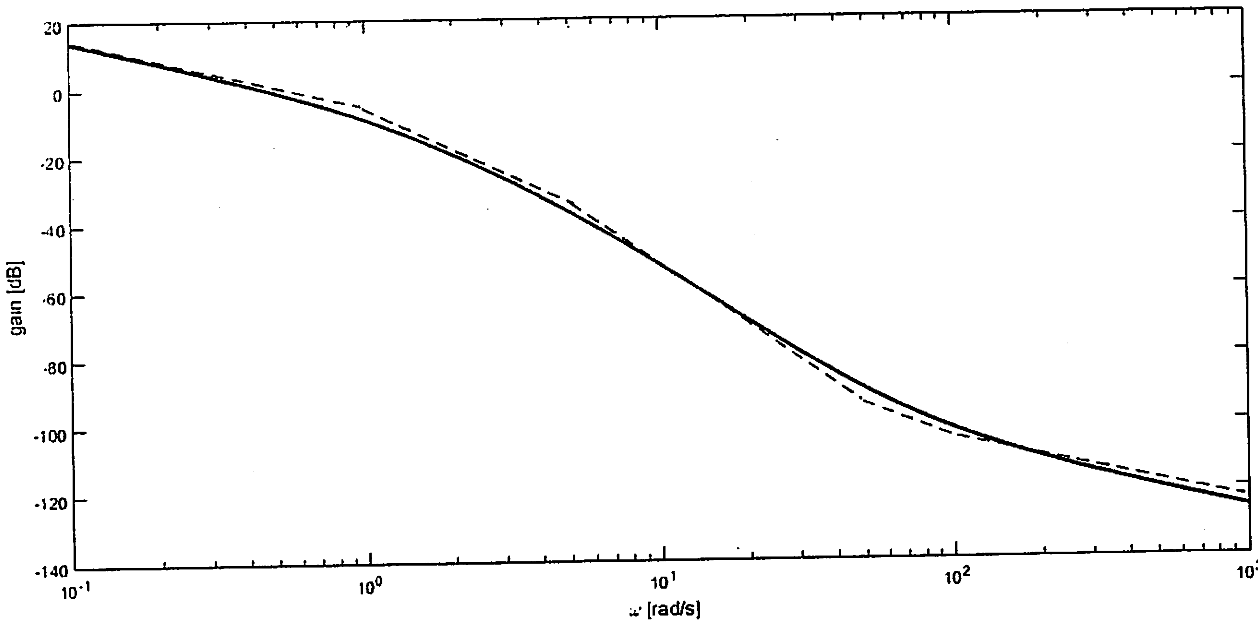
      ii. What is the time delay T that will destabilize the system?

   (f) What is the steady state error of the closed-loop system to unit step input for the gain K in (d)?

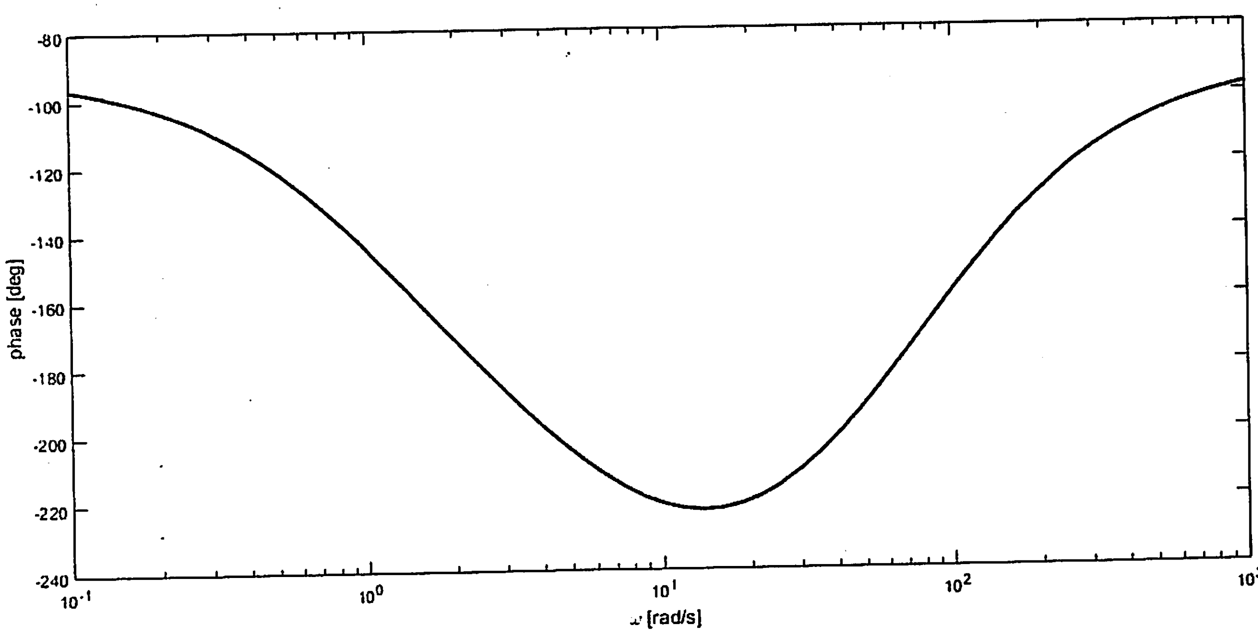
   (g) What is the steady state error of the closed-loop system to unit ramp input for the gain K in (d)?

   (h) What is the steady state response of the open-loop system $G(s)$ to the input

$$r(t) = \begin{cases} 2 \cdot sin^2 t - 5 \cdot sin\,(10t + \pi) & \text{for} \quad t \geq 0 \\ 0 & \text{for} \quad t < 0 \end{cases}$$



20
0
-20
-40
-60
-80
-100
-120
-140
gain [dB]
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
ω [rad/s]



-80
-100
-120
-140
-160
-180
-200
-220
-240
phase [deg]
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
ω [rad/s]

Ph.D. Screening Examination Problem – AME 541

USC Aerospace and Mechanical Engineering Department

January 17, 2016

Linear Time-Invariant Mechanical System. The cartoon in Fig. 1 shows two linear springs with constants k_1 and k_2 attached to masses m_1 and m_2, respectively, where the position of m_1 is d_1 and the position of m_2 is d_2. The right end of the spring with constant k_1 is attached to the left end of the spring with constant k_2 at **position** u, which is arbitrarily externally determined by an experimenter. The system of coordinates is defined so that when $u = d_1 = d_2 = 0$, the springs are at their natural lengths.

(a) Write the **two** equations of motion describing the mechanical system.

(b) Defining $u(t)$ as the input, $y(t) = \begin{bmatrix} d_1(t) & d_2(t) \end{bmatrix}^T$ as the output and the state as $x(t) = \begin{bmatrix} d_1(t) & \dot{d}_1(t) & d_2(t) & \dot{d}_2(t) \end{bmatrix}^T$, find a **four-dimensional** state-space realization describing the system, i.e., find matrices $\{A, B, C, D\}$ for the equations

$$\dot{x}(t) = Ax(t) + Bu(t),$$
$$y(t) = Cx(t) + Du(t).$$

(c) Explain the notion of internal asymptotic stability for the generic *linear time-invariant* (LTI) case.

(d) For $m_1 = 1, m_2 = 1, k_1 = 1$ and $k_2 = 1$, determine if the system described by the state-space realization in (b) is asymptotically stable or not. Clearly, justify your answer.

Hint: Notice that

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ j & -j \end{bmatrix} \begin{bmatrix} j & 0 \\ 0 & -j \end{bmatrix} \begin{bmatrix} 0.5 & -0.5j \\ 0.5 & 0.5j \end{bmatrix}.$$

(e) Explain the notion of controllability associated with the generic pair $\{A, B\}$.

(f) For $m_1 = 1, m_2 = 1, k_1 = 1$ and $k_2 = 1$, determine if the system described by the state-space realization in (b) is controllable or not. Clearly, justify your answer.

(g) Explain the notion of observability associated with the generic pair $\{C, A\}$.

(h) For $m_1 = 1, m_2 = 1, k_1 = 1$ and $k_2 = 1$, determine if the system described by the state-space realization in (b) is observable or not. Clearly, justify your answer.

(i) For $m_1 = 1, m_2 = 1, k_1 = 1$ and $k_2 = 1$, using the Kâlmân decomposition and the controllability/observability duality theorem, find a minimal realization of the system defined according to $\{A, B, C, D\}$ in (b). Write each step of your solution.

(j) For $m_1 = 1, m_2 = 1, k_1 = 1$ and $k_2 = 1$, find the transfer matrix $\hat{g}(s)$ associated with the state-space realization in (b).

(k) Find the impulse response $g(t)$ associated with the transfer matrix $\hat{g}(s)$ in (j).

(m) Is $g(t)$ in (k) absolutely integrable? Clearly, justify your answer.

(n) Is $\hat{g}(s)$ BIBO stable? Justify your statement using your answer for item (m).

(o) For $m_1 = 1, m_2 = 1$ and generic k_1, k_2, show that the system in (b) is controllable **if and only if**

$$\text{rank}\begin{bmatrix} k_1 & -k_1^2 \\ k_2 & -k_2^2 \end{bmatrix} = 2.$$

(p) Assuming that the system in (b) satisfies the controllability condition in (o), how would you design a state feedback controller in order to change the natural frequencies of the system by the use of feedback? Explain and draw a block diagram.

(q) For $m_1 = 1, m_2 = 1, k_1 = 1, k_2 = 1$, initial state $x(0) = \begin{bmatrix} 0.1 & 0 & -0.1 & 0 \end{bmatrix}^T$ and input $u(t) = 0.1 \cdot \mathbf{1}(t)$, find the output $y(t)$, for $t \geqslant 0$.

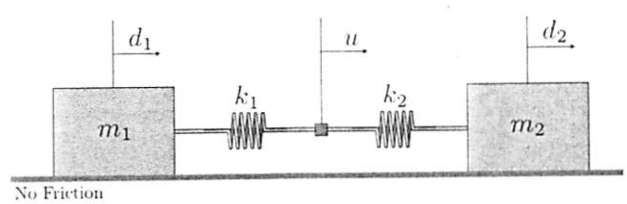


Figure 1: Two-Mass Mechanical System.

Appendix. The following is a list of useful definitions, formulas, identities and facts (use them wisely):

- The solution to the continuous-time *linear time-invariant* (LTI) state-space equations

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t), \\ y(t) &= Cx(t) + Du(t), \end{aligned}$$

is given by

$$x(t) = e^{At}x(0) + \int_0^t e^{A(t-\tau)}Bu(\tau)d\tau$$

and

$$\begin{aligned} y(t) &= Ce^{At}x(0) + C\int_0^t e^{A(t-\tau)}Bu(\tau)d\tau + Du(t) \\ &= Ce^{At}x(0) + \{g * u\}(t) \\ &= Ce^{At}x(0) + \{u * g\}(t). \end{aligned}$$

- The step function (also known as the Heaviside function) is defined as

$$\mathbb{1}(t) = \begin{cases} 0, & \text{for } t < 0; \\ 1, & \text{for } t \geqslant 0. \end{cases}$$

- The transfer matrix, $\hat{g}(s)$, associated with a state-space realization $\{A, B, C, D\}$ can be computed as

$$\hat{g}(s) = C\,(sI - A)^{-1}\,B + D.$$

- The inverse of a generic 2×2 matrix is given by

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

- The inverse of a generic nonsingular block diagonal square matrix is given by

$$A^{-1} = \begin{bmatrix} A_1 & 0 & \cdots & \cdots & 0 \\ 0 & A_2 & 0 & \cdots & 0 \\ 0 & 0 & A_3 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \vdots & \ddots & A_n \end{bmatrix}^{-1} = \begin{bmatrix} A_1^{-1} & 0 & \cdots & \cdots & 0 \\ 0 & A_2^{-1} & 0 & \cdots & 0 \\ 0 & 0 & A_3^{-1} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \vdots & \ddots & A_n^{-1} \end{bmatrix}.$$

- The controllability matrix associated with the pair $\{A, B\}$ is computed as

$$\mathcal{C} = \begin{bmatrix} B & AB & \cdots & A^{n-1}B \end{bmatrix}.$$

- The observability matrix associated with the pair $\{C, A\}$ is computed as

$$\mathcal{O} = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix}.$$

Control Theory: Screening Examination
Spring 2016

January 22, 2016

The examination is closed-books and closed-notes.

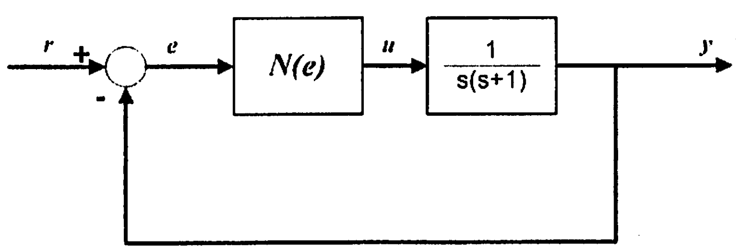
1. Given a system in a standard form as shown in the figure with the nonlinearity as shown $K_1 > 0$, and $K_2 > 0$. and $K_1 \neq K_2$. Assume that the input $r = 0$.

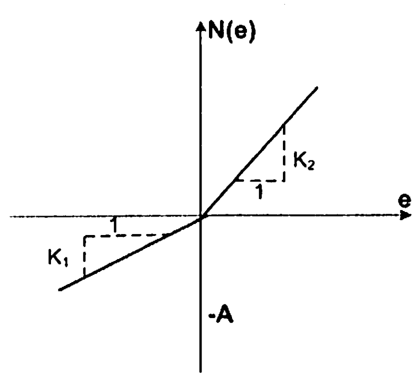
   (a) Can you establish asymptotic stability of the origin of the closed-loop system by linearization ? If the answer is **yes**, show only the steps needed to establish asymptotic stability (no computation is needed). If the answer is **no**, explain why it cannot be done.

   (b) Show that the the origin of the closed-loop system is globally asymptotically stable for all $K_1 > 0$ and $K_2 > 0$.

   (c) Is the Aizerman's conjecture satisfied for this system? Explain.

   (d) Assume that the gains are functions of time, $K_1(t) \geq 0$, $K_2(t) > 0$ and $K_1(t) > K_2(t)$ for all t. Find the conditions for global asymptotic stability of the origin.





Design

USC/AME

PH.D. SCREENING EXAM - TECHNICAL FIELD: **DESIGN**

JANUARY 22, 2016

Note: If you take this topic as major: Complete **3** problems.
If you take this topic as minor: Complete **2** problems from 2 different classes.

Conceptual Design (AME-410)

Problem 1: Complete the following two questions:

Question (1): What is the process of systematic design? Explain why it is helpful and its limitations.

Question (2): Select one product from the following list:

Cloth washing machine, Personal vehicle, Advanced alarm clock

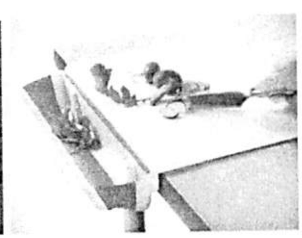
(a) Identify the overall function of the product and draw a functional structure of the product. The functional structure should have at least five functions. Briefly explain the function structure.

(b) Based on the functional structure, develop an unconventional design concept for the product by following the morphology chart approach; describe how your design concept is different from the existing product.

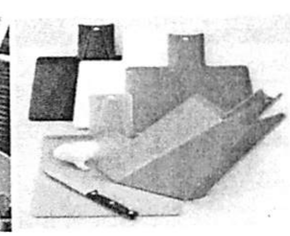
Advanced Mechanical Design (AME-503)

Problem 2: A kitchen chopping board (shown in Figure A) must have a flat surface for cutting the food. However, with a completely flat board, it is hard to hold the chopped food and put them directly into the cooking pot (see Figure C). An alternative design for both cutting and holding is shown in Figure B. Another new chopping board (shown in Figure D) is now getting more popular.


(A)


(B)


(C)


(D)

a) Please show the design matrix of a traditional flat-surface chopping board shown in Figure A in terms of the two functional requirements identified above.
b) Please show the design matrix of the alternative chopping board shown in Figure B in terms of the two functional requirements identified above.

c) Please show the design matrix of the new chopping board shown in Figure D in terms of the two functional requirements identified above.
d) Please use the Axiomatic Design Theory to explain the reason why the modern chopping board (Figure D) is a better design than both the traditional chopping board (Figure A) and the alternative chopping board (Figure B)
e) Please use the TRIZ Approach to formulate a physical contradiction in current design of the traditional chopping board (Figure A).
f) Please use one of the four TRIZ separation principles to resolve the above physical contradiction as a way to improve the traditional chopping board (Figure A).

Engineering Information Modeling (AME-505)

Problem 3: Complete the following two questions:

Question (1): Prepare a list of objects that you would expect each of the following systems to handle. Explain briefly why you need these objects.

a) A system for supporting Project Management
b) A program to support a Household Cleaning Robot

Question (2): Develop an object model (i.e., a use-case diagram and a class diagram with a paragraph explanation for each) for the following problem.

"A system for automating components inventory management is needed. The system should keep track of various types of components, their brief description of properties, their amount, and their locations in the storage. After receiving a component order form from a requester, the system should search for the component, identify the location of the requested component, and send a robotic mover to the location. The robotic mover then will move to the location, pick up the needed component, and move it to the requestor. The system should also produce daily report of the current inventory and send it to the inventory manager."

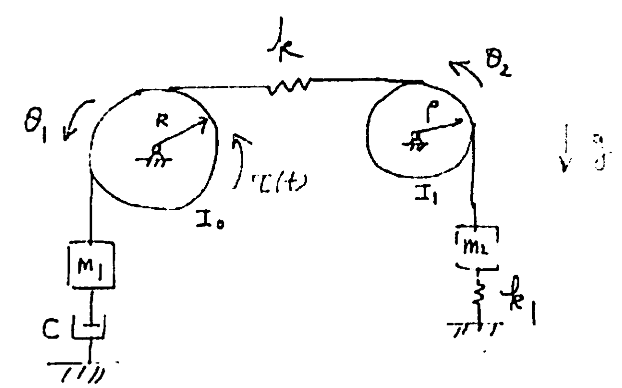
Dynamics and Vibrations

Ph.D. Screening Examination Problems for Vibrations & Dynamics

January 2016

Problem 1 (AME 420 level)

Consider the 2-DOF system in the figure below, where two disks are connected by a spring, each disk is connected to a lumped mass by an unstretchable rope, and the left disk is subject to a torque $\tau(t)$. (10)



(a) Derive the governing equations of motion for the system. You must consider gravity.

(b) Consider the following non-dimensional parameters

$$2I_o = I_1 = 10, \quad m_1 = m_2 = 2$$

$$R = \rho = 2, \quad k = 90, \quad k_1 = 15$$

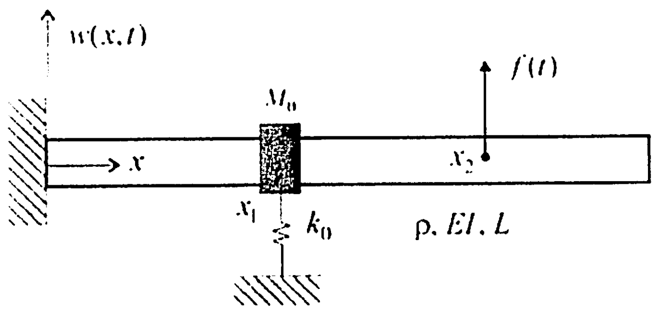
Compute the natural frequencies of the system.

(c) Ignore gravity. Consider the same non-dimensional parameters as in (b). Given a sinusoidal toque $\tau(t) = 10\sin(3t)$, determine the steady-state rotation angle θ_2 of the right disk.

Problem 2 (AME 521 level)

In the figure, a transversely vibrating uniform cantilever beam carries a lumped mass M_0 at $x = x_1$ ($0 < x_1 < L$), which is constrained by a spring of coefficient k_0 and has the same motion as the beam. A point-wise external force $f(t)$ is applied to the beam at $x = x_2$.

(10)



(a) Write down all the governing equations for the combined beam system and identify the essential and natural boundary conditions of the beam.

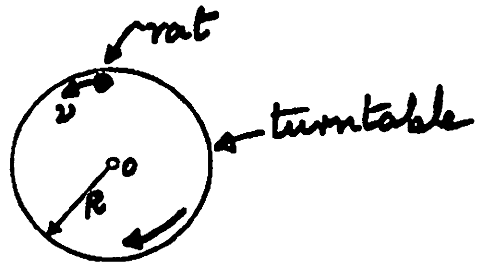
(b) Estimate the first natural frequency of the combined beam. You can use whatever method you prefer.

(c) By the assumed-modes method, obtain a discretized model of the coupled mass-beam as follows

$$[M]\{\ddot{q}\}+[K]\{q\}=\{F\}.$$

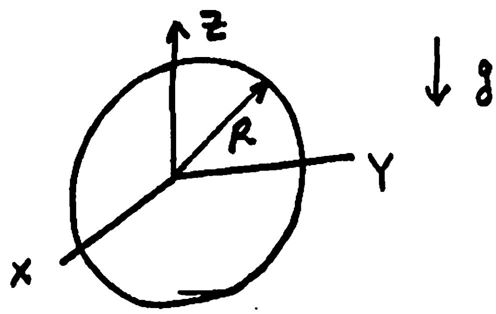
You need to give a sequence of admissible functions. You don't need to evaluate the integrals in the discretization process, but you must show the formulas for calculating the elements of matrices $[M]$ and $[K]$.

3. (a) A laboratory rat (mass, m) is trained to run around the rim of a turntable of radius R at constant speed v. The turntable, whose surface is in the horizontal plane, starts from rest and speeds up with a constant angular acceleration, c. Find the force acting on the rat at any time time, t, after the turntable starts. The turntable rotates about its pivot, O, in a direction opposite to that in which the rat runs around it. See Figure below.



(5)

3. (b) A point particle of mass *m* is constrained to move on the surface of a sphere in a gravitational field with gravity acting vertically downwards. Write the equations of motion for this particle as it moves. (5)

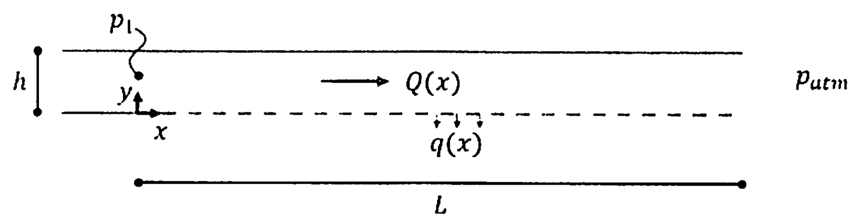


Fluid Dynamics

AME 530a Dynamics of Incompressible Fluids

2016 Screening Exam

An incompressible fluid of density ρ and viscosity μ flows through a rectangular channel of length L, width w (into the page) and height h. Note that the channel height is small, such that $h \ll L$ and $h \ll w$. Fluid emerges from the channel outlet to the atmosphere, which is at pressure p_{atm}. The pressure at the inlet is p_1.



The bottom plate of the channel is *slightly* porous and some fluid leaks out through it. The local leakage volume flow rate, $q(x)$ per unit area of plate, depends on the pressure difference according to:

$$q(x) = k[p(x) - p_{atm}],$$

where the constant k reflects the permeability of the plate, and $p(x)$ is the local pressure inside the channel. In all that follows, you can assume that this is a steady, inertia-free (i.e. low Reynolds number) flow, and that the leakage velocity is small compared to the local horizontal velocity.

a) Obtain a differential equation that relates the gradient in the horizontal volume flow rate dQ/dx at any point x to the local pressure $p(x)$.

b) Starting with the x-component of the Navier-Stokes equations and making appropriate assumptions, obtain a differential equation that predicts the pressure distribution inside the channel as a function of x, the channel geometry, fluid properties, and atmospheric pressure p_{atm}. State the boundary conditions necessary to solve this differential equation.

   [Hint: Estimate the horizontal velocity profile in terms of dp/dx and integrate this profile to get Q. You'll also need the solution from part (a)].

c) Based on the form of the ordinary differential equation obtained in part (b), identify via scaling arguments the length-scale l_p over which the pressure drops substantially. Qualitatively describe what happens to the pressure distribution and flow rate at the limits where $l_p \gg L$ and $l_p \ll L$.

Spring 2016 - Compressible Flow Question

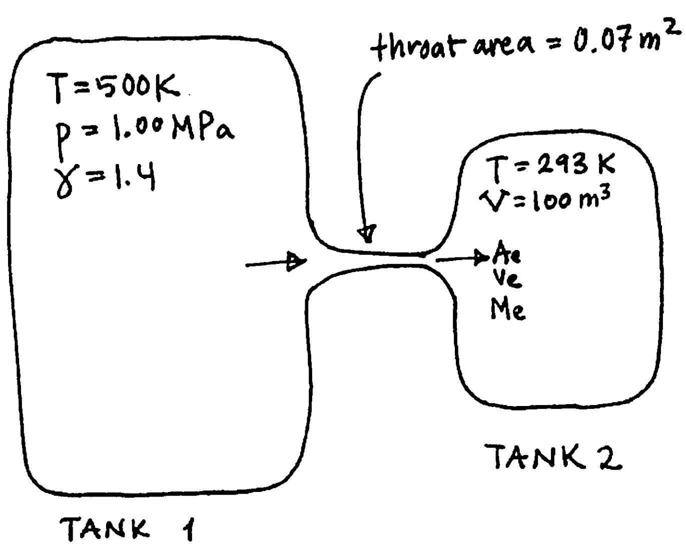
For full marks, all assumptions must be stated and the solution must have the appropriate units shown. For full marks, (a), (b), and (c) all have to be solved. Tables are attached on the following pages.

The two tanks in the figure below are attached via a converging diverging nozzle that is designed to have a Mach number of $M_e = 2.00$ at the exit plane (assuming the flow remains nearly isentropic). The flow travels from Tank 1 to Tank 2, where Tank 1 is much, much larger than Tank 2. The volume of Tank 2 is 100m^3.

(a) Find the exit area A_e and the back pressure p_b that will allow the system to operate at design conditions.

(b) As time goes on, the back pressure will grow, since the second tank slowly fills up with more air. Since Tank 1 is much, much larger than Tank 2, the flow in the nozzle will remain constant until the time when a normal shock wave appears at the exit plane. At what back pressure will this occur?

(c) If Tank 2 is held at constant temperature, $T = 293K$, estimate how long time it will take for the flow to go from design conditions to the condition of question (b) with a shock wave at the exit plane.



Computational Fluid Dynamics

Consider one-dimensional heat equation for temperature $T(x,t)$

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}.$$

The equation is to be solved numerically on the interval $[0,L]$ using uniform grid with N points (mesh size $\Delta x = L/(N-1)$) and time step Δt. The temperature at both ends is fixed at all times, $T(0,t) = T_1 = A$, and $T(L,t) = T_N = B$.

1. (4 points) Discretize this equation using **forward** differencing for the time derivative, i.e. using values of T at time levels n and $n+1$, and for the spatial derivative use **central** difference formulas weighted between old and new time levels, n and $n+1$, respectively

$$\left[\frac{\partial^2 T}{\partial x^2}\right] = a\left[\frac{\partial^2 T}{\partial x^2}\right]^{n+1} + b\left[\frac{\partial^2 T}{\partial x^2}\right]^{n},$$

   where a and b are the weights.

2. (8 points) Prove that the scheme is consistent only if $a+b=1$.

3. (3 points) For the remaining parts of this problem consider a special case $a=1/2$ and $b=1/2$. Write an algorithm to solve the discretized equations, i.e., the algebraic equations with unknown variables T^{n+1} at time step $n+1$ on the l.h.s. and known variables at time step n on the r.h.s. Make sure to write separately two equations that involve prescribed boundary conditions.

4. (2 points) Is this an explicit or an implicit numerical scheme? Do you know its name?

5. (2 points) What is the order of the truncation error for this scheme?

6. (4 points) Using von Neumann's method find the stability condition for this scheme.

7. (2 points) In general, what information is needed to determine if a numerical scheme is convergent? Do you have enough information from previous parts of the problem to make this determination for the scheme in question?

Useful formulas:

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$1 - \cos\theta = 2\sin^2\tfrac{\theta}{2}$$